Decision Tree Learning-Inspired Dynamic Variable Ordering for the Weighted CSP

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Agenda

- The Weighted Constraint Satisfaction Problem (WCSP)
- Branch-and-Bound Search and Dynamic Variable Ordering (DVO)
- Our Decision-Tree Learning Inspired Dynamic Variable Ordering
- Experimental Evaluation
- Conclusion

Executive Summary

- Branch-and-bound search has been the state of the art paradigm for solving the WCSP.
- Dynamic variable ordering (DVO) is a critical component of branch-and-bound search.
- Our newly proposed DVO algorithms, inspired by decision tree learning, have shown superior performance in our preliminary experiments.

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The Weighted Constraint Satisfaction Problem: Motivation

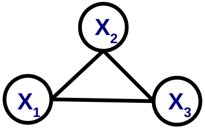
Many real-world problems can be solved using the WCSP:

- RNA motif localization (Zytnicki et al. 2008)
- Communication through noisy channels using Error Correcting Codes in Information Theory (Yedidia et al. 2003)
- Medical and mechanical diagnostics (Milho et al. 2000; Muscettola et al. 1998)
- Energy minimization in Computer Vision (Kolmogorov 2005)
- ...

Weighted Constraint Satisfaction Problem (WCSP)

- N variables $\underline{x} = \{X_1, X_2, \ldots, X_N\}$.
- Each variable X_i has a discrete-valued domain D_i.
- M weighted constraints $\{E_{s_1}, E_{s_2}, \ldots, E_{s_M}\}$.
- Each constraint E_s specifies the weight for each combination of assignments of values to a subset s of the variables.
- Find an optimal assignment of values to these variables so as to minimize the total weight: $E(\underline{x}) = \sum_{i=1}^{M} E_{s_i}(\underline{x}_{s_i})$.
- Known to be NP-hard.

WCSP Example on Boolean Variables



X_1	
0	0.7
1	0.2

X_2	
0	0.3
1	0.8

X_3	
0	0.1
1	1.0

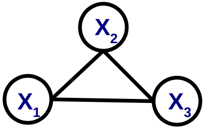
X_1 \ X_2	0	1
0	0.5	0.6
1	0.7	0.3

X_2 \ X_3	0	1
0	0.6	1.3
1	1.0	1.1

X_1 \ X_3	0	1
0	0.4	0.9
1	0.7	0.8

$$E(X_1, X_2, X_3) = E_1(X_1) + E_2(X_2) + E_3(X_3) + E_{12}(X_1, X_2) + E_{13}(X_1, X_3) + E_{23}(X_2, X_3)$$

WCSP Example: Evaluate the Assignment $X_1 = 0, X_2 = 0, X_3 = 1$



X_1	
0	0.7
1	0.2

X_2	
0	0.3
1	0.8

X_3	
0	0.1
1	1.0

X_1 \ X_2	0	1
0	0.5	0.6
1	0.7	0.3

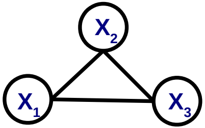
X_2 \ X_3	0	1
0	0.6	1.3
1	1.0	1.1

X_1 \ X_3	0	1
0	0.4	0.9
1	0.7	0.8

$$E(X_1 = 0, X_2 = 0, X_3 = 1) = 0.7 + 0.3 + 1.0 + 0.5 + 1.3 + 0.9 = 4.7$$

(This is not an optimal solution.)

WCSP Example: Evaluate the Assignment $X_1 = 1, X_2 = 0, X_3 = 0$



X_1	
0	0.7
1	0.2

X_2	
0	0.3
1	0.8

X_3	
0	0.1
1	1.0

X_1 \ X_2	0	1
0	0.5	0.6
1	0.7	0.3

X_2 \ X_3	0	1
0	0.6	1.3
1	1.0	1.1

X_1 \ X_3	0	1
0	0.4	0.9
1	0.7	0.8

$$E(X_1 = 1, X_2 = 0, X_3 = 0) = 0.2 + 0.3 + 0.1 + 0.7 + 0.6 + 0.7 = 2.6$$

This is an optimal solution. Using brute force, it requires exponential time

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Branch-and-Bound Search

Search by assigning value to one variable at a time until the optimal solution is found. Backtrack when needed.

Each search node consists of

- an assignment of value to a subset of variables and the total weight of constraints between all assigned variables w_a
- the total weight of currently best solution $w^\dagger$

At each search node:

1. Choose a variable X_k assign a value x_k to it. (Dynamic Variable Ordering)
2. Enforce local consistency.
3. Compute w_a.
4. If all variables have been assigned and $w_a < w^\dagger$, then $w^\dagger := w_a$ and backtrack.
5. If $w_a \geq w^\dagger$, backtrack.
6. Go to 1 (next search node).

Dynamic Variable Ordering (DVO): Example of Two Search Orders

A 3-variable WCSP instance:

X_1 \ X_2	0	1
0	400	300
1	200	1

(a) Constraint C_1

X_2 \ X_3	0	1
0	1	3
1	2	4

(b) Constraint C_2

$X_1 \to X_2 \to X_3$, first 0 then 1

$X_1 = 0$ $\quad w_a = 0, w^\dagger = \infty$

$X_1 = 0, X_2 = 0$ $\quad w_a = 400, w^\dagger = \infty$

$X_1 = 0, X_2 = 0, X_3 = 0$ $\quad w_a = 401, w^\dagger = 401$

$X_1 = 0, X_2 = 0, X_3 = 1$ $\quad w_a = 402, w^\dagger = 401$

$X_1 = 0, X_2 = 1$ $\quad w_a = 300, w^\dagger = 7$

$X_1 = 0, X_2 = 1, X_3 = 0$ $\quad w_a = 302, w^\dagger = 302$

...

$X_1 = 1, X_2 = 1, X_3 = 0$ $\quad w_a = 3, w^\dagger = 3$

$X_1 = 1, X_2 = 1, X_3 = 1$ $\quad w_a = 5, w^\dagger = 5$

Found the optimal solution by visiting 14 search nodes.

Dynamic Variable Ordering (DVO): Example of Two Search Orders

A 3-variable WCSP instance:

X_1 \ X_2	0	1
0	400	300
1	200	1

(a) Constraint C_1

X_2 \ X_3	0	1
0	1	3
1	2	4

(b) Constraint C_2

$X_1 \to X_2 \to X_3$, first 1 then 0

$X_1 = 1$	$w_a = 0, w^\dagger = \infty$
$X_1 = 1, X_2 = 1$	$w_a = 1, w^\dagger = \infty$
$X_1 = 1, X_2 = 1, X_3 = 1$	$w_a = 5, w^\dagger = 5$
$X_1 = 1, X_2 = 1, X_3 = 0$	$w_a = 3, w^\dagger = 3$
$X_1 = 1, X_2 = 0$	$w_a = 200, w^\dagger = 3$
$X_1 = 0$	$w_a = 0, w^\dagger = 3$
$X_1 = 0, X_2 = 1$	$w_a = 300, w^\dagger = 3$
$X_1 = 0, X_2 = 0$	$w_a = 400, w^\dagger = 3$

Found the optimal solution by visiting only 8 search nodes.

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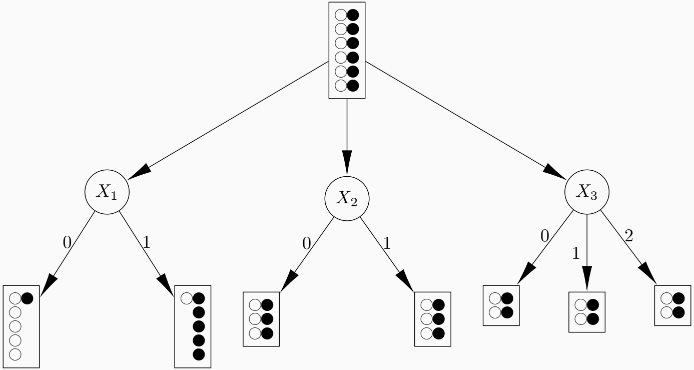
Intuition

X_1 \ X_2	0	1
0	1	2
1	102	3

(a) Constraint C_1

X_2 \ X_3	0	1	2
0	1	2	3
1	102	3	101

(b) Constraint C_2



(c) Search tree

Measurement

The measurement can be based on sampling and computing:

- sdr the standard deviation, or
- rr the range of weights in the samples (i.e., the maximum weight minus the minimum weight).

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Setup

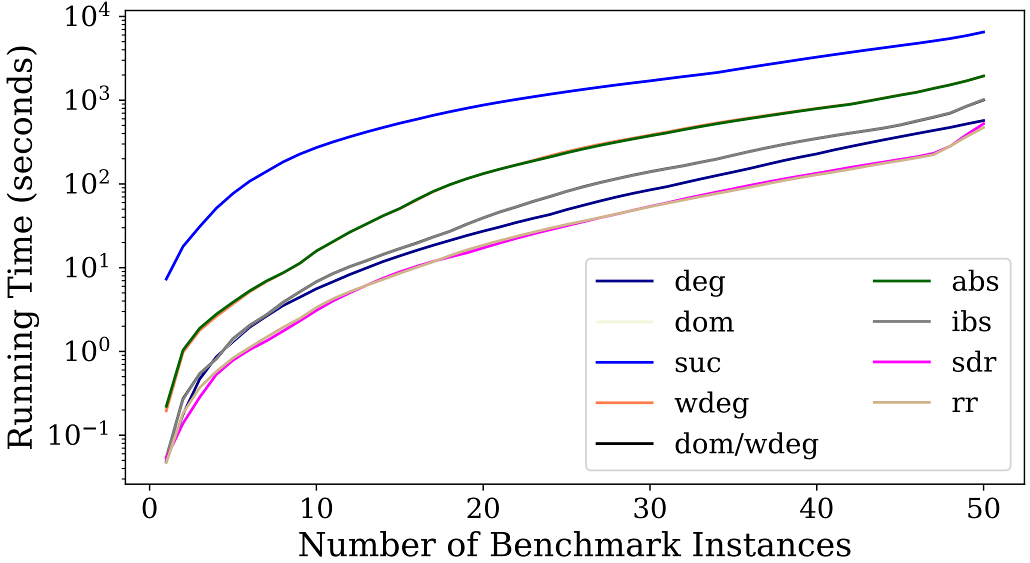
- Our algorithms: sdr, rr, sdr-bound, rr-bound
- Competitors
  - deg, dom, suc ((Heras et al. 2006))
  - wdeg, dom/wdeg ((Boussemart et al. 2004))
  - abs ((Michel et al. 2012))
  - ibs ((Refalo 2004))
  - sdr-inv, sdr-inv-bound, rr-inv, rr-inv-bound (Use the reverse of the measurements of sdr, sdr-bound, rr, rr-bound)

Setup

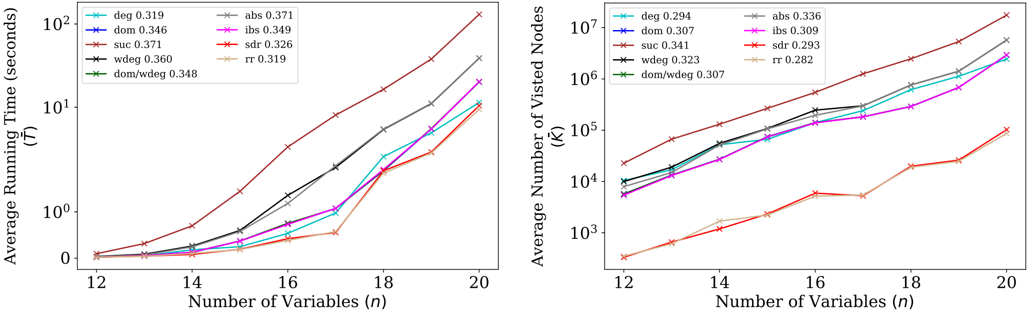
- Benchmarks:
  - (Hurley et al. 2016)
    - Limited choice to at most 25 variables and domain size no more than 6.
    - Only 6 instances satisfy the condition.
  - Due to the scarcity of real-world instances, we also created random instances:
    - Create n variables,
    - add a constraint between every two variables with probability $p = 0.1$,
    - randomly assign weights from 1 to 100.
    - We generated 50 such instances for each n ranging from 12 to 20.

Real-World Instances

Name	ff1	j4	l4	q5	q3	q4
$\|\mathcal{X}\|$	2	28	8	25	25	25
$\|\mathcal{C}\|$	3	196	32	185	185	185
$\hat{D}$	5	2	6	5	3	4
sdr	$31/3 \cdot 10^{-4}$s	833/0.27s	101/0.05s	391,065/4042s	-/48h	-/48h
sdr-bound	$31/3 \cdot 10^{-4}$s	637/1.60s	11/0.04s	6/0.94s	-/48h	-/48h
rr	$31/3 \cdot 10^{-4}$s	801/2.16s	109/0.16s	1100/9.95s	-/48h	-/48h
rr-bound	$31/1 \cdot 10^{-2}$s	665/1.71s	11/0.08s	6/0.97s	-/48h	-/48h
inv-sdr	$31/2 \cdot 10^{-4}$s	5491/1.64s	179/0.05s	429,005/4984s	-/48h	-/48h
inv-sdr-bound	$31/2 \cdot 10^{-4}$s	667/1.80s	8/0.08s	6/0.94s	-/48h	-/48h
inv-rr	$31/2 \cdot 10^{-4}$s	5943/11.97s	429/0.29s	14,677/44.78s	-/48h	-/48h
inv-rr-bound	$31/2 \cdot 10^{-4}$s	659/1.58s	10/0.08s	6/0.94s	-/48h	-/48h
deg	$31/1 \cdot 10^{-4}$s	3225/1.26s	187/0.04s	27,834,834/48,163s	-/48h	-/48h
dom	$31/9 \cdot 10^{-5}$s	8623/5.24s	331/0.08s	-/48h	-/48h	-/48h
suc	$31/9 \cdot 10^{-5}$s	3491/1.72s	606/0.12s	7,718,377/8867s	-/48h	-/48h
wdeg	$31/9 \cdot 10^{-5}$s	8623/5.37s	203/0.15s	-/48h	-/48h	-/48h
dom/wdeg	$31/9 \cdot 10^{-5}$s	8623/5.29s	331/0.08s	-/48h	-/48h	-/48h
abs	$31/2 \cdot 10^{-4}$s	3173/2.73s	404/0.33s	1,814,781/911s	-/48h	-/48h
ibs	$31/1 \cdot 10^{-4}$s	7045/4.53s	236/0.08s	-/48h	-/48h	-/48h



Running Time (seconds)
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
0
10
20
30
40
50
Number of Benchmark Instances
deg
dom
suc
wdeg
dom/wdeg
abs
ibs
sdr
rr


deg 0.319
dom 0.346
suc 0.371
wdeg 0.360
dom/wdeg 0.348
abs 0.371
ibs 0.349
sdr 0.326
rr 0.319
Average Running Time (seconds) ($\bar{T}$)
Number of Variables (n)
deg 0.294
dom 0.307
suc 0.341
wdeg 0.323
dom/wdeg 0.307
abs 0.336
ibs 0.309
sdr 0.293
rr 0.282
Average Number of Visted Nodes ($\bar{K}$)
Number of Variables (n)

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- Dynamic Variable Ordering (DVO) algorithms can be critical in WCSP solving.
- We created two new DVO algorithms, inspired by decision tree learning.
- In our preliminary experiments, they have shown more superior performance compared with current state-of-the-art algorithms.
- Future Work: Integrate our new DVO algorithms with state-of-the-art WCSP solvers like toulbar2 (Hurley et al. 2016).

References i

Frederic Boussemart, Fred Hemery, Christophe Lecoutre, and Lakhdar Sais. “Boosting systematic search by weighting constraints”. In: *the European Conference on Artificial Intelligence*. 2004, pp. 146–150.

Federico Heras and Javier Larrosa. “Intelligent variable orderings and re-orderings in DAC-based solvers for WCSP”. In: *Journal of Heuristics* 12.4 (2006), pp. 287–306. DOI: `10.1007/s10732-006-8248-z`.

Barry Hurley, Barry O’Sullivan, David Allouche, George Katsirelos, Thomas Schiex, Matthias Zytnicki, and Simon de Givry. “Multi-language evaluation of exact solvers in graphical model discrete optimization”. In: *Constraints* 21.3 (2016), pp. 413–434.

Vladimir Kolmogorov. *Primal-dual Algorithm for Convex Markov Random Fields*. Tech. rep. MSR-TR-2005-117. Microsoft Research, 2005.

Laurent Michel and Pascal Van Hentenryck. “Activity-Based Search for Black-Box Constraint Programming Solvers”. In: *the International Conference on Integration of Artificial Intelligence and Operations Research Techniques in Constraint Programming*. 2012, pp. 228–243. DOI: `10.1007/978-3-642-29828-8_15`.

References ii

Isabel Milho, Ana Fred, Jorge Albano, Nuno Baptista, and Paulo Sena. "An Auxiliary System for Medical Diagnosis Based on Bayesian Belief Networks". In: *Portuguese Conference on Pattern Recognition*. 2000.

Nicola Muscettola, P. Pandurang Nayak, Barney Pell, and Brian C. Williams. "Remote Agent: to boldly go where no {AI} system has gone before". In: *Artificial Intelligence* 103.1–2 (1998), pp. 5–47.

Philippe Refalo. "Impact-Based Search Strategies for Constraint Programming". In: *the International Conference on Principles and Practice of Constraint Programming*. 2004, pp. 557–571. DOI: 10.1007/978-3-540-30201-8_41.

Jonathan S Yedidia, William T Freeman, and Yair Weiss. "Understanding belief propagation and its generalizations". In: *Exploring Artificial Intelligence in the New Millennium* 8 (2003), pp. 236–239.

Matthias Zytnicki, Christine Gaspin, and Thomas Schiex. "DARN! A Weighted Constraint Solver for RNA Motif Localization". In: *Constraints* 13.1 (2008), pp. 91–109.