In this paper, we develop the message passing based linear-time and linear-space MVC algorithm (MVC-MPL) for solving the minimum vertex cover (MVC) problem. MVC-MPL is based on heuristics derived from a theoretical analysis of message passing algorithms in the context of belief propagation. We show that MVC-MPL produces smaller vertex covers than other linear-time and linear-space algorithms.

### III. Warning Propagation for the MVC Problem

Proposed and analyzed by Weigt and Zhou (2006). There is a message of either 0 or 1 along each direction of each edge. Assuming \( u \) and \( v \) are two adjacent vertices, \( u \) sends a message of 1 to "warn" \( v \) to indicate that \( u \) will not be selected in the vertex cover. Otherwise, \( u \) sends \( v \) a message of 0.

- A vertex is in the vertex cover iff it has at least one incoming message of 1.
- Assuming independence of the probabilities of messages, a given vertex can be chosen to be in or not in the vertex cover based on the probability of having at least one incoming message of 1.

\[ \text{Algorithm 1: MVC-MPL} \]

```plaintext
Function MVC-MPL(G = (V,E))
Input: G: The graph to find an MVC for.
Output: A VC of G:
1. Initialize VC = ∅ and IS = ∅;
2. \( c := \text{average degree of vertices in } G; \)
3. \( p_i := 1 - W(c)/c; \) // Fraction of zero messages
4. while \( 3e \in V \backslash (VC \cup IS) \) do
5. \( k(v) := |\{ u \in \partial v \mid u \notin VC \}|; \) // Number of adjacent vertices
6. Draw a random real number \( r \) uniformly at random from \([0,1]\);
7. if \( r < p_i \) then
8. Add \( v \) to IS;
9. else
10. Add all \( u \in \partial v \) to VC;
11. Add \( v \) to VC;
12. return VC;
```

V. Experimental Results

| Instance                  | \(|V|\) | \(|E|\) | MVC-MPL | MVC-L | MVC-2 | MVC-MPL | MVC-L | MVC-2 |
|---------------------------|--------|--------|---------|-------|-------|---------|-------|-------|
| bn-human-BNU-1-0025664-1bg | 696,338 | 143,158,340 | 647,568 | 650,013 | 686,776 | 724 | 925 | 1,101 |
| soc-livejournal2008        | 4,031,137 | 27,933,063 | 2,148,197 | 2,205,385 | 2,591,926 | 893 | 971 | 731 |
| soc-livejournal2007        | 5,363,201 | 79,023,143 | 3,127,083 | 3,623,388 | 4,908,058 | 1,200 | 1,363 | 1,492 |
| tech-assiette              | 1,696,415 | 11,695,299 | 624,654 | 695,988 | 819,280 | 253 | 252 | 193 |
| tech-ip                    | 2,250,498 | 21,644,715 | 68,525 | 122,670 | 132,640 | 681 | 497 | 176 |
| web-baidu-bake             | 2,141,330 | 17,794,639 | 745,685 | 784,284 | 1,061,712 | 406 | 300 | 218 |
| web-living-ding            | 1,984,484 | 14,869,484 | 713,449 | 743,685 | 1,011,712 | 336 | 309 | 130 |

### References


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