A Distributed Logical Filter for Connected Row Convex Constraints

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Executive Summary

The Kalman filter and its distributed variants are successful methods in state estimation in stochastic models. We develop the analogues in domains described using constraints.

Agenda

What is Filtering and What is Its Motivation

The Connected Row Convex (CRC) Filter

Distributed Connected Row Convex (CRC) Filter

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Motivation

Navigation system (Zarchan et al. 2015)

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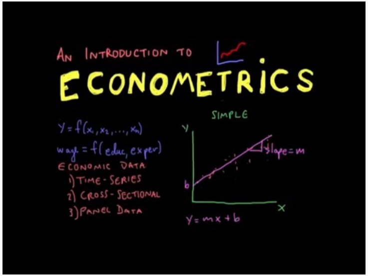
`https://commons.wikimedia.org/wiki/File:`

`Navigation_system_on_a_merchant_ship.jpg`

Econometrics (Schneider 1988)



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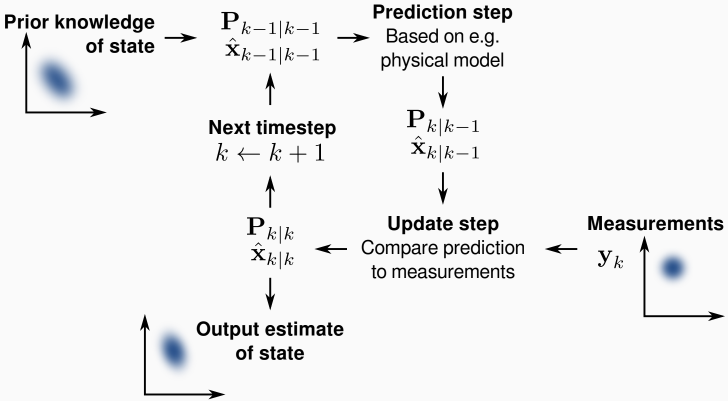
Filtering

In a partially observable or uncertain environment, an agent often needs to maintain its belief state (a representation of its knowledge about the world) based on

- What are the beliefs at previous time steps?
- What does the agent observe at the current time step?

Filtering denotes any method whereby an agent updates its belief state.

Example: The Kalman Filter



Kalman filter (Kalman 1960)

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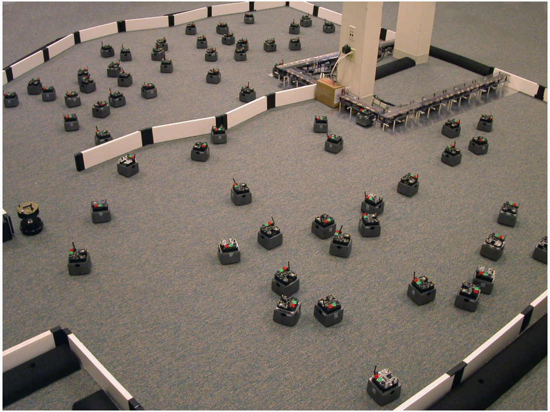
Logical Filter

A logical filter applies to domains that are described using logical formulae or constraints.

Here, we are interested in the connected row convex (CRC) filter (Kumar et al. 2006), a logical filter that uses CRC constraints.

Motivation: Multi-Robot Localization

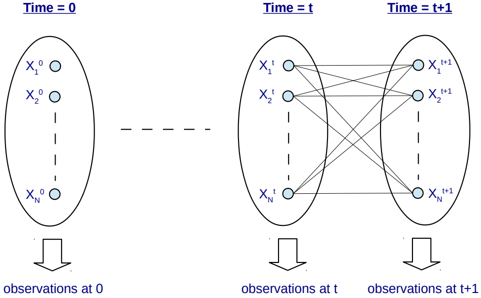


by James McLurkin. Retreived from: `https://people.csail.mit.edu/jamesm/project-MultiRobotSystemsEngineering.php`

Constraint Satisfaction Problems (CSPs)

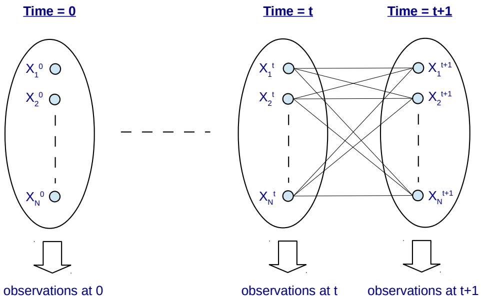
- N variables $\mathcal{X} = \{X_1, X_2, \ldots, X_N\}$.
- Each variable X_i has a discrete-valued domain $\mathcal{D}(X_i)$.
- M constraints $\mathcal{C} = \{C_1, C_2, \ldots, C_M\}$.
- Each constraint C_i specifies allowed and disallowed assignments of values to a subset $S(C_i)$ of the variables.
- Find an assignment a of values to these variables such that all constraints allow it.
- Known to be NP-hard.

A Filter Based on Constraints: Framework



- Observations at t are modeled as constraints on the variables at t.
- Transitions from t to $t+1$ are modeled as the constraints between variables at t and $t+1$.
- At each time step $t+1$, the belief state is defined by all allowed assignments of values to variables at $t+1$ that satisfy observation constraints at $t+1$, and have a consistent extension to variables at t (with Markovian assumption).

A Filter Based on Constraints: Framework



But...

in general, determining the existence of a consistent extension to the previous time step requires looking further back.

We'd like to have compact information.

Solution:

Connected Row Convex (CRC) Constraints

The Connected Row Convex (CRC) Constraint

X_i \ X_j	d_{j1}	d_{j2}	d_{j3}	d_{j4}	d_{j5}
d_{i1}	0	1	0	0	0
d_{i2}	1	1	0	0	0
d_{i3}	0	1	1	0	1
d_{i4}	0	0	1	0	0
d_{i5}	0	0	1	0	0

(a) ✓

X_i \ X_j	d_{j1}	d_{j2}	d_{j3}	d_{j4}	d_{j5}
d_{i1}	0	1	0	0	0
d_{i2}	1	0	0	0	0
d_{i3}	0	0	1	1	1
d_{i4}	0	0	1	0	0
d_{i5}	0	0	1	0	0

(b) ✗

‘1’: Allowed assignment ‘0’: Disallowed assignment

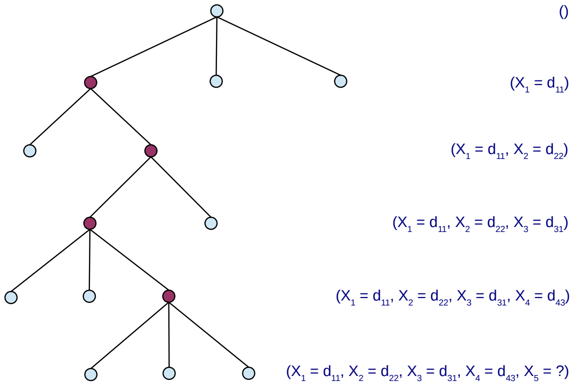
Row convex constraint: All ‘1’s in each row are consecutive

CRC constraint: Row convex + The ‘1’s in any two successive rows/columns intersect or are consecutive after removing empty rows/columns

The Connected Row Convex (CRC) Constraint

- Path consistency: For any consistent assignment of values to any two variables X_i and X_j, there exists a consistent extension to any other variable X_k.
- After enforcing path consistency on constraint networks with only CRC constraints, all constraints are still CRC. This is not true for row convex constraints.

The Connected Row Convex (CRC) Constraint

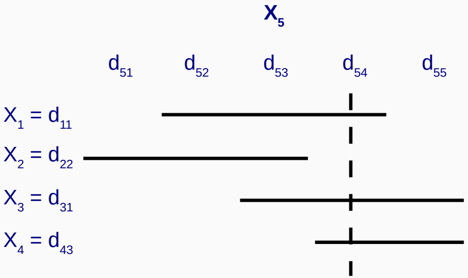


CSP search tree

The Connected Row Convex (CRC) Constraint

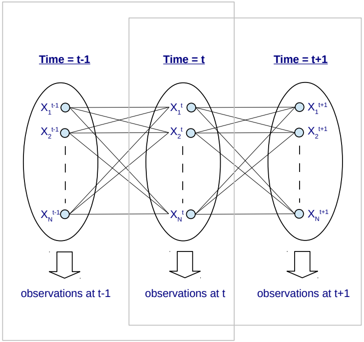
X_5

	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}
$X_1 = d_{11}$	0	1	1	1	0
$X_2 = d_{22}$	1	1	1	0	0
$X_3 = d_{31}$	0	0	1	1	1
$X_4 = d_{43}$	0	0	0	1	1

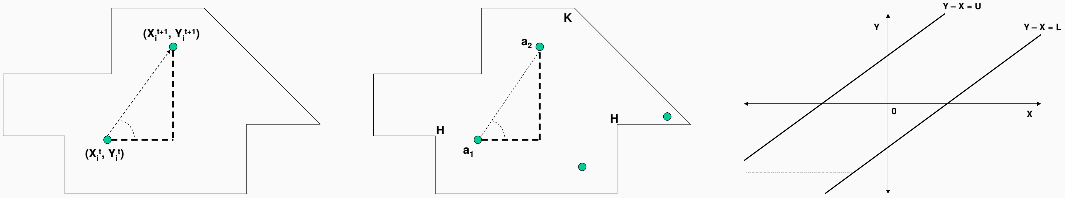


Row convexity implies global consistency in path consistent constraint networks

The Connected Row Convex (CRC) Filter



If all constraints are CRC, enforcing path consistency between every two consecutive time steps t and $t+1$ leads to new CRC constraints between variables at $t+1$. These CRC constraints contain all information of consistent assignments.



(a) Each robot estimate its own movement.

(b) Each robot estimate its distances from other robots.

(c) The constraints are CRC.

(Kumar et al. 2006, Fig. 18)

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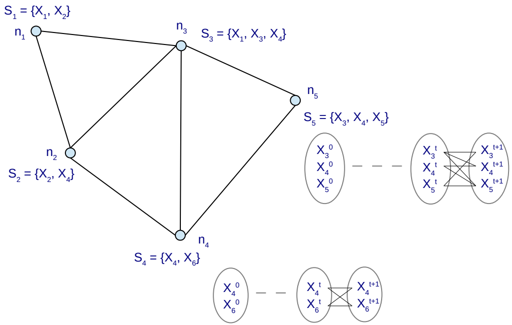
The Distributed Kalman Filter

The distributed version of the Kalman filter has been successful in state estimation in wireless sensor networks (Rao et al. 1993), including large scale systems (Olfati-Saber 2007).

What about a distributed CRC filter?

Distributed Connected Row Convex (CRC) Filter



- Each agent is in charge of a subset of variables, and all constraints involving those variables.
- The system evolves using distributed path consistency.
- Improving distributed path consistency algorithms is key to the success of the CRC filter.

Conclusion

- Filtering denotes any method whereby an agent updates its belief state.
- The Kalman filter is well known in stochastic models.
- A logical filter is a filter that uses logical formulae or constraints.
- The CRC filter is the long-pursued logical analogue of the Kalman filter.
- The distributed CRC filter is a logical analogue of the distributed Kalman filter and requires distributed path consistency.

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